9. COMPARISON OF THEORY AND MEASUREMENTS

Robert J. Achatz and Roger A. Dalke[1]

It is important to compare theoretical calculations with measurements whenever possible to validate results. Such validation is usually based on comparing salient features of measured results with corresponding features based on theoretical predictions.

Theoretical results given in Section 3 predict that 1) fixed time-base dithering will attenuate spectral lines (relative to the non-dithered periodic signal) and also introduce a continuous spectrum with a shape similar to the pulse spectrum, 2) periodically repeating the dithered signal will introduce spectral lines at the dither signal repetition frequency, and 3) statistics obtained from filtered UWB signals will be approximately Gaussian when the bandwidth is less than the PRR. Note that baseband Gaussian signal statistics yield Rayleigh amplitude statistics.

Device D measurements, summarized in Section 8.3.4, are for a 25% fixed dithered UWB device. Measurements include a 10 MHz PRR mode with 100% gating. These measurement results were compared with theoretical predictions given in Section 3. A synopsis of the comparison is given in this section.

Fixed time-base dithering features

Theoretical results for the 50% fixed time-base dithered, 10 MHz PRR PSD are presented in Section 3.2.1. These results predict that at frequencies of several hundred MHz and higher, spectral lines are only 10-20 dB above the continuous spectrum and hence should not be observable when the receiver bandwidth significantly exceeds 20 dB Hz. In addition, the continuous spectrum should have a shape corresponding to the pulse spectrum.

The pulse shape of Device D was measured with the full bandwidth test described in Section 5.2. The emission spectra was measured with the spectrum analyzer test described in Section 6. The spectrum analyzer test for a 25% fixed dithered, 10 MHz PRR UWB signal is summarized in Figure 8.53. These emission spectra measurements have the same shape as the pulse spectrum measured by the full bandwidth test in Figure 8.49 as predicted. Also, spectral lines at the PRR intervals were not evident in the emission spectra measurements as predicted by theory.

---

[1]The authors are with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, CO 80305.

Dither signal repetition features

Theoretical results presented in Section 3.2.2 show that for repeated fixed dithered signals, spectral lines should be observed at frequencies corresponding to the signal repetition period. Figure 8.59 shows an emission spectrum measurement of a 25 % fixed dither, 10 MHz PRR UWB signal with a dither repetition frequency of 10 kHz. The measurement shows the spectral lines due to the repeating dither signal at 10 kHz intervals as predicted.

Band Limited UWB signal APD features

Theoretical results for the band limited 50% fixed dithered, 10 MHz PRR, UWB signal PSD were presented in Section 3.3. These results show that when the UWB signal passes through a receiver filter that is well below the PRR, the signal statistics are approximately Gaussian. As the filter bandwidth increases, the shape of the density function deviates from Gaussian. This deviation is measured by a statistic called excess which is a function of filter bandwidth as shown in Figure 3.5. Excesses near zero are indicative of an underlying Gaussian process. The excess factor deviates from zero at approximately 3 MHz bandwidth. This trend continues and at 10 MHz the value of the excess is significantly greater than zero.

The APD of for a 25 % fixed dithered, 10 MHz PRR APD shows the same non-Gaussian trend for bandwidths greater than 3 MHz where the statistics become increasingly impulsive. When the UWB signal is passed through filters with bandwidths of 1 MHz or less measurements show that the signal statistics are approximately Gaussian as predicted by theory.

APPENDIX A. TUTORIAL ON USING AMPLITUDE PROBABILITY DISTRIBUTIONS TO CHARACTERIZE THE INTERFERENCE OF ULTRAWIDEBAND TRANSMITTERS TO NARROWBAND RECEIVERS

Robert J. Achatz[1]

A.1 Introduction

The *amplitude probability distribution function* (APD) is used in radio engineering to describe signal amplitude *statistics*. The APD and its corresponding graph, shown in Figure A.1, succinctly express the probability that a signal amplitude exceeds a threshold. For example, the APD in Figure A.1 shows that the signal amplitude rarely exceeds high voltages. Statistics such as percentiles, deciles, and the median can be read directly from the APD. Other statistics such as average power can be computed with the APD.

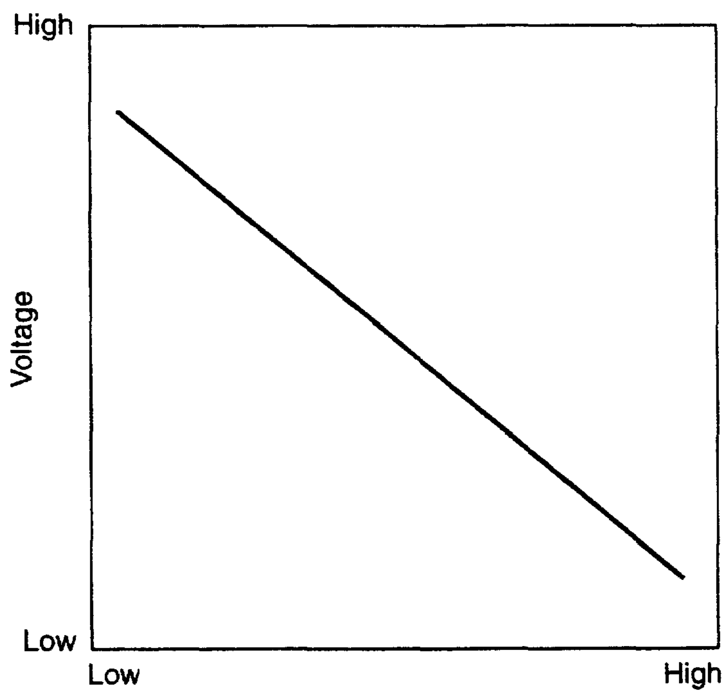

Figure A.1. Amplitude probability distribution.

The "signal" the APD characterizes is often noise or interference. For example, APDs are commonly used to characterize the amplitude statistics of *non-Gaussian* noise produced by

[1]The author is with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, CO 80305.

lightning or unintentional emissions from man-made electrical or electronic devices. Numerous studies have shown that average noise power alone cannot predict the performance of receivers operating in non-Gaussian noise. APD statistics are needed for accurate predictions.

Today, many radio engineers are unfamiliar with the APD and its applications. This is because most modern receivers are designed to operate in bands with (zero-mean) *Gaussian* noise which is completely characterized by the average noise power statistic alone. Consequently, APD statistics are not needed for more accurate predictions.

Recently, federal spectrum regulators have been asked to allow emissions from *ultrawideband* (UWB) transmitters to overlay bands licensed to services that use *narrowband* receivers. Critics have charged that UWB transmitters may cause interference to 'victim' narrowband receivers. The amplitude statistics of this potential interference are dependent upon the specifications of the UWB signal and the band limiting filter in the narrowband receiver. The APD can be used to characterize this interference and correlate UWB signal and band limiting filter specifications to narrowband receiver performance.

The purpose of this tutorial is to introduce basic APD concepts to radio engineers and spectrum regulators who have not previously used the APD. It is hoped that these concepts will provide a firm basis for discussions on regulation of UWB transmitters. Emphasis is placed on understanding features likely to be found in band limited UWB signal APDs. These features are demonstrated with "tutorial" APDs of Gaussian noise, sinusoid (continuous wave) signals, and periodically pulsed sinusoid signals. Although the audience is intended to be broad, a limited number of mathematic expressions are used to avoid the ambiguity found in everyday language.

A.2 Signal Amplitude Characterization

A.2.1 APD Fundamentals

A *bandpass signal* is a signal whose bandwidth is much less than the *center frequency*. Bandpass signals are expressed mathematically as

$$s(t) = A(t)\cos(2\pi f_c + \theta(t)),$$

where A(t) is the baseband amplitude, $\theta(t)$ is the baseband phase, and f_c is the center frequency. The amplitude and phase define the *complex baseband signal*, $A(t)e^{j\theta(t)}$, whose spectrum is centered about 0 Hz.

The amplitude is always positive and is considered to be a *random variable*, A, when characterized by an APD. Formally, a new random variable, $\underline{A}_n$, is present at each sampling instant. The set $\{A_1, A_2, \ldots A_N\}$ is called the *random sample* of the random variable A if each random variable is independent and identically distributed. Realizations or values of the random sample are denoted by the set $\{a_1, a_2, \ldots a_N\}$.

Associated with every random variable is a *probability density function* (PDF). The discrete PDF expresses the *probability* that a random variable "A" will have a realization equal to "a_i":

$$p(a_i) = P(A=a_i) ,$$

where $P()$ is the probability of its argument. PDF values are positive and the area under a PDF is equal to 1.0.

The *cumulative distribution function* CDF expresses the probability that a random variable "A" will have a realization less than or equal to "a":

$$c(a) = P(A \leq a) .$$

The discrete CDF is obtained by integrating the discrete PDF

$$c(a) = \sum_i p(a_i) ,$$

for all a_i less than or equal to a. CDF values range from 0.0 to 1.0.

Radio engineers are generally more concerned about how often a noise or interference amplitude exceeds a threshold. Thus they often prefer to use the complement of the CDF (CCDF) or APD. The APD function expresses the probability that a random variable "A" will have a realization greater than "a":

$$cc(a) = P(A>a) .$$

The discrete APD is obtained by subtracting the discrete CDF from 1.0.

$$cc(a) = 1.0\text{-}c(a) .$$

For clarification, Figure A.2 shows graphs of the discrete PDF, CDF, and APD for the random sample realizations:

$$\{a_1, a_2, \ldots a_{10}\} = \{1,2,3,3,1,4,4,3,4,3\} \text{ volts.}$$

The discrete PDF is estimated from the histogram. By convention, the axes of the APD are oriented differently from the axes of the CDF and PDF.

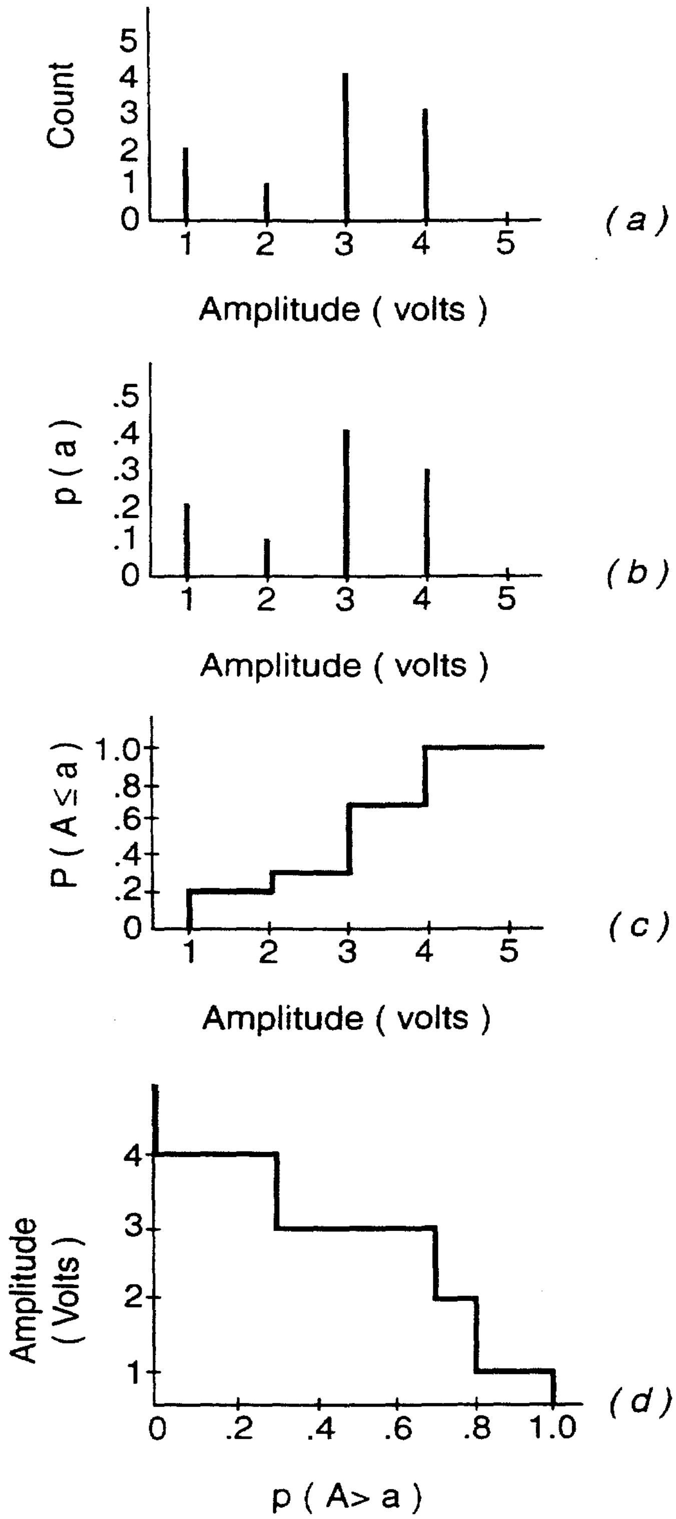

Figure A.2. Example histogram (a), probability density function (b), cumulative distribution function (c), and amplitude probability distribution function (d).

A.2.2 Statistic fundamentals

Statistics are functions that operate on the random sample. The *statistic value* is the result of a statistic operating on *random sample values*. Figure A.3 illustrates these relationships. Common statistical functions are percentile, mean or average, and root mean square (RMS). *First-order* statistics, addressed in this tutorial, assume the random variables are independent and identically distributed. *Second-order* statistics, not addressed in this tutorial, measure the correlation between these random variables. *Stationary* statistics are independent of time whereas *non-stationary* statistics are functions of time. Noise and interference amplitude statistics are non-stationary in many cases. Thus radio engineers sometimes measure the statistics of the amplitude statistics such as the median average noise power.

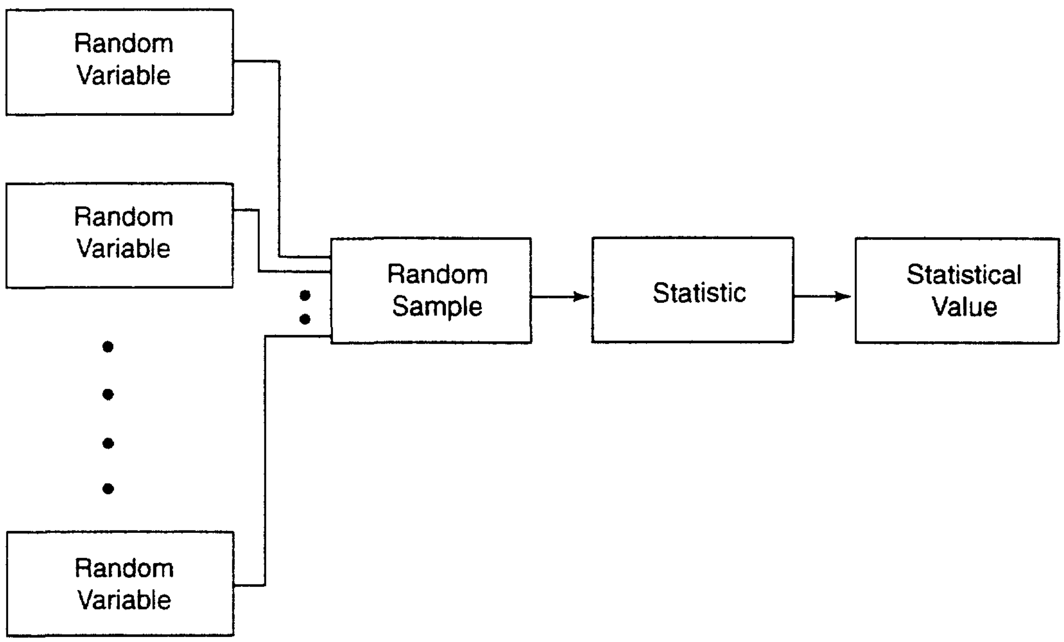

Figure A.3. Language of statistics.

Percentile amplitude statistics can be read directly from the APD. Peak and median amplitude statistics are the most widely used percentile statistics. The peak statistic is sometimes arbitrarily defined by the amplitude that is exceeded 0.0001% of the time:

$$V_p = cc^{-1}(0.000001) ,$$

where $a = cc^{-1}(P(A>a))$. The median statistic is defined by the amplitude that is exceeded 50% of the time:

$$V_{median} = cc^{-1}(0.5) .$$

The mean and RMS statistics are determined directly from the random sample values. The mean statistic is defined by:

$$V_{mean} = \frac{1}{N}\sum_n a_n ,$$

where N is the number of samples. The mean-logarithm statistic is defined by:

$$V_{mean-log} = \frac{1}{N}\sum_n \log_{10}(a_n) ,$$

and the RMS statistic is defined by:

$$V_{RMS} = \sqrt{\frac{1}{N}\sum_n a_n^2} .$$

The discrete APD and its corresponding discrete PDF can be used to calculate the mean and RMS statistics if the random sample values are no longer available. The choice of histogram bin size may affect the accuracy of these statistics. In this case the mean statistic is defined by:

$$V_{mean} = \sum_i a_i p(a_i) ,$$

where a_i represents a discrete PDF value. The mean logarithm statistic is defined by:

$$V_{mean-log} = \sum_i \log_{10}(a_i) p(a_i) ,$$

and the RMS statistic is defined by:

$$V_{RMS} = \sqrt{\sum_i a_i^2 p(a_i)} .$$

As a reference, statistical values for the tutorial PDF presented in section 2.1 are 4.0, 3.0, 2.8, 0.4, and 3.0 for the peak, median, mean, mean logarithm, and RMS statistics.

A.2.3 Graphing the APD

The APD of Gaussian noise is of particular interest to radio engineers because it is encountered in many practical applications. The amplitude of Gaussian noise is *Rayleigh distributed.* A Rayleigh distributed random variable is represented by a straight, negatively-sloped line on a *Rayleigh graph*. Figure A.4 shows the APDs of Gaussian and non-Gaussian noise on a Rayleigh graph.

The Rayleigh graph displays probability on the x-axis and amplitude on the y-axis. The probability is scaled by the function

$$x = 0.5\log_{10}(-\ln(P(A>a))) ,$$

and converted to percent to represent the "*percent (of samples or time) exceeding ordinate.*" The amplitude in volts is converted to units of power such as dBW, i.e. scaled by the function

$$y = 20\log_{10}(A) \,,$$

or alternatively it is displayed in dB relative to a standard noise power density or noise power.

Figure A.4 shows the statistics of Gaussian noise on a Rayleigh graph. Gaussian noise average power or RMS voltage corresponds to the power or voltage that is exceeded 37% of the time. Gaussian noise peak voltage is approximately 10 dB above RMS voltage. Gaussian noise average voltage, median voltage, and average-logarithm voltage are approximately 1dB, 2 dB, and 2.5 dB below RMS voltage respectively.

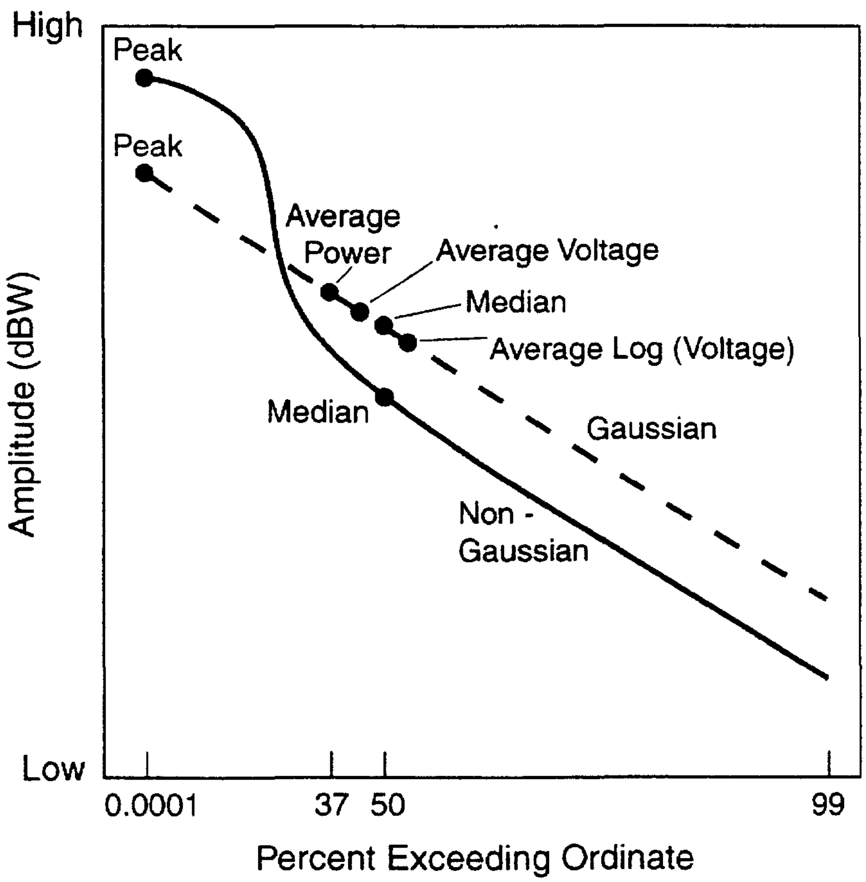

Figure A.4. Gaussian noise and non-Gaussian noise APDs plotted on a Rayleigh graph.

A.3 Tutorial APDs

A.3.1 Random Noise

Band limited random noise, i.e. the random noise present after a band limiting filter, is a random-amplitude and random-phase bandpass "signal" defined by

$$n(t) = A(t)\cos(2\pi f_c + \theta(t)) .$$

Band limited random noise is represented in the frequency domain by a *power spectral density* (PSD) in units of watts/Hz. The random noise "signal," amplitude, and PSD are shown in Figure A.5.

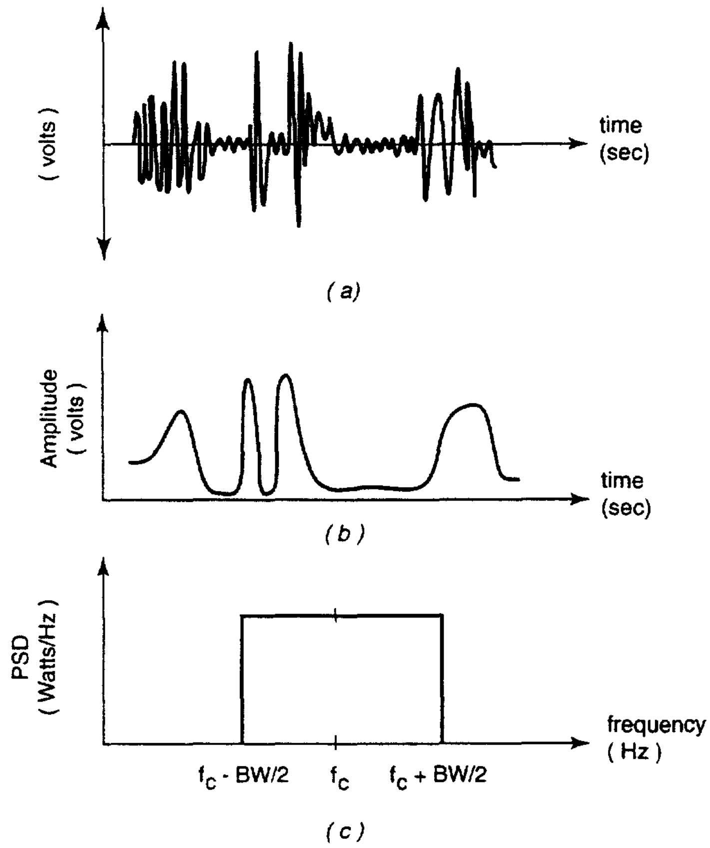

Figure A.5. Random noise (a), amplitude (b), and power spectral density (c).

Band limited noise average power is computed from the noise PSD

$$P = \int_{f_c - BW/2}^{f_c + BW/2} N(f)df \quad ,$$

where N(f) is the noise PSD in units of watts/Hz. Band limited *white* noise power density is constant over the band limiting filter bandwidth. As a result, the average noise power is directly proportional to the filter bandwidth and the RMS amplitude is directly proportional to the square root of bandwidth. This is sometimes referred to as the "$10\log_{10}$ bandwidth" rule. The amplitude of band limited Gaussian noise is Rayleigh distributed. Figure A.6 shows the APD of band limited white-Gaussian noise (WGN) for two different bandwidths on a Rayleigh graph.

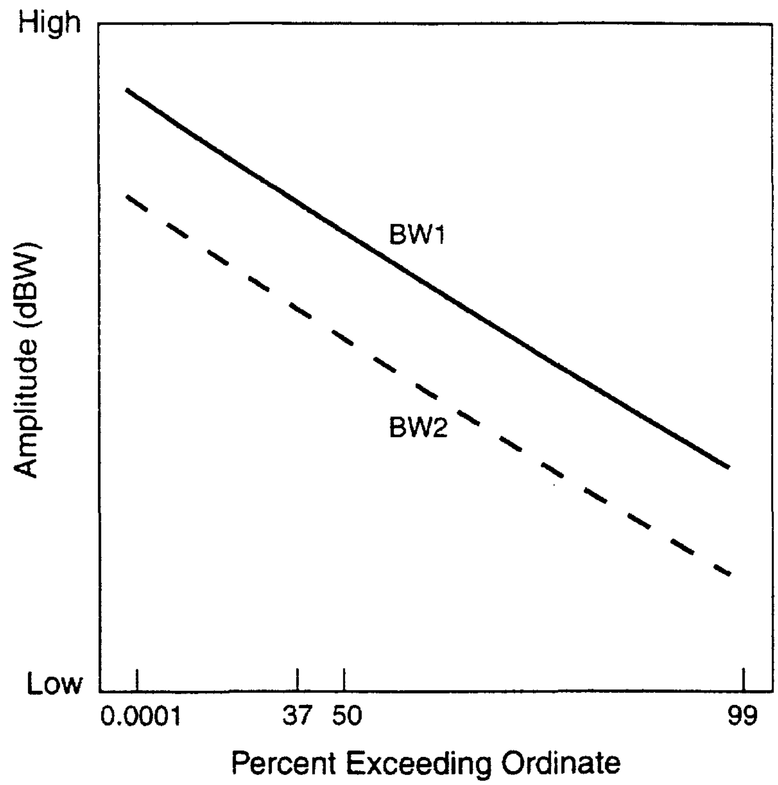

Figure A.6. Bandlimited Gaussian noise APDs with two different bandwidths. BW1 is greater than BW2.

A.3.2 Sinusoid Signal

The sinusoid (continuous wave) signal is a narrowband, constant amplitude and constant phase signal. It is defined by

$$s(t) = A\cos(2\pi f_c + \theta) .$$

The signal, signal amplitude, and amplitude spectrum are shown in Figure A.7. The APD of the sinusoid signal is a flat line from the lowest to the highest percentile on a Rayleigh graph. Changing the receiver center frequency can change the amplitude of the sinusoid signal.

Widening the bandwidth of a receiver filter in the presence of noise causes the statistics to be *Rician*. Rician statistics are dependent on the ratio of the sinusoid power to the noise power. The Rician APD corresponds to the sinusoid signal APD when noise is absent and the Rayleigh APD when the signal is absent. Sinusoid, Rician, and Rayleigh APDs are depicted in Figure A.8.

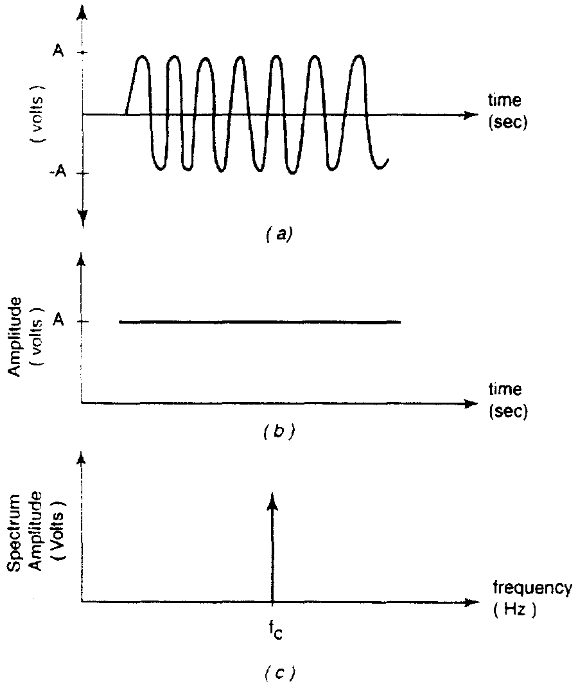

Figure A.7. Sinusoid signal (a), signal amplitude (b), and amplitude of the signal spectrum (c).

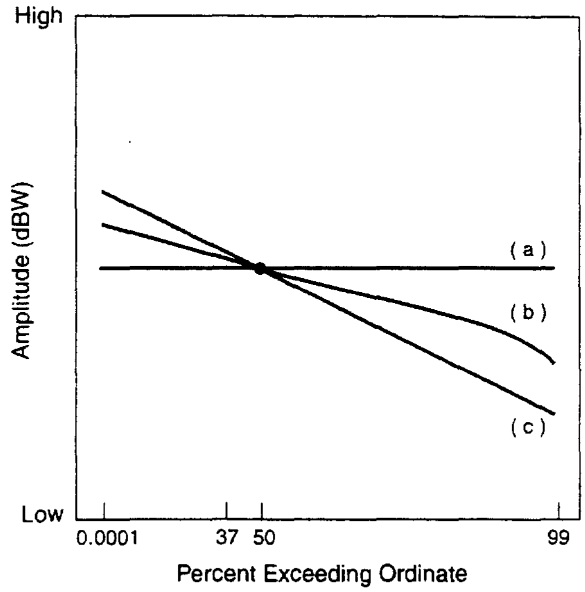

Figure A.8. Sinusoid signal APD without (a) and with (b) random noise. Sinusoid signal with random noise has Rician amplitude statistics. The Gaussian noise APD (c) is included for reference.

A.3.3 Periodically Pulsed Sinusoid

The periodically pulsed sinusoid is a deterministic, time-varying amplitude and constant phase signal defined by

$$s(t) = A(t)cos(2\pi f_c + \theta) .$$

The amplitude varies between 'on' and 'off' pulse states. The 'on' duration is the *pulse width* (PW) and the repetition rate of pulses is the *pulse repetition rate* (PRR) or the pulse repetition frequency (PRF). Amplitude spectrum *lines* are spaced at the PRR. Amplitude spectrum *nulls* are spaced by the reciprocal of the PW. The signal, signal amplitude, and amplitude of the signal spectrum are shown in Figure A.9.

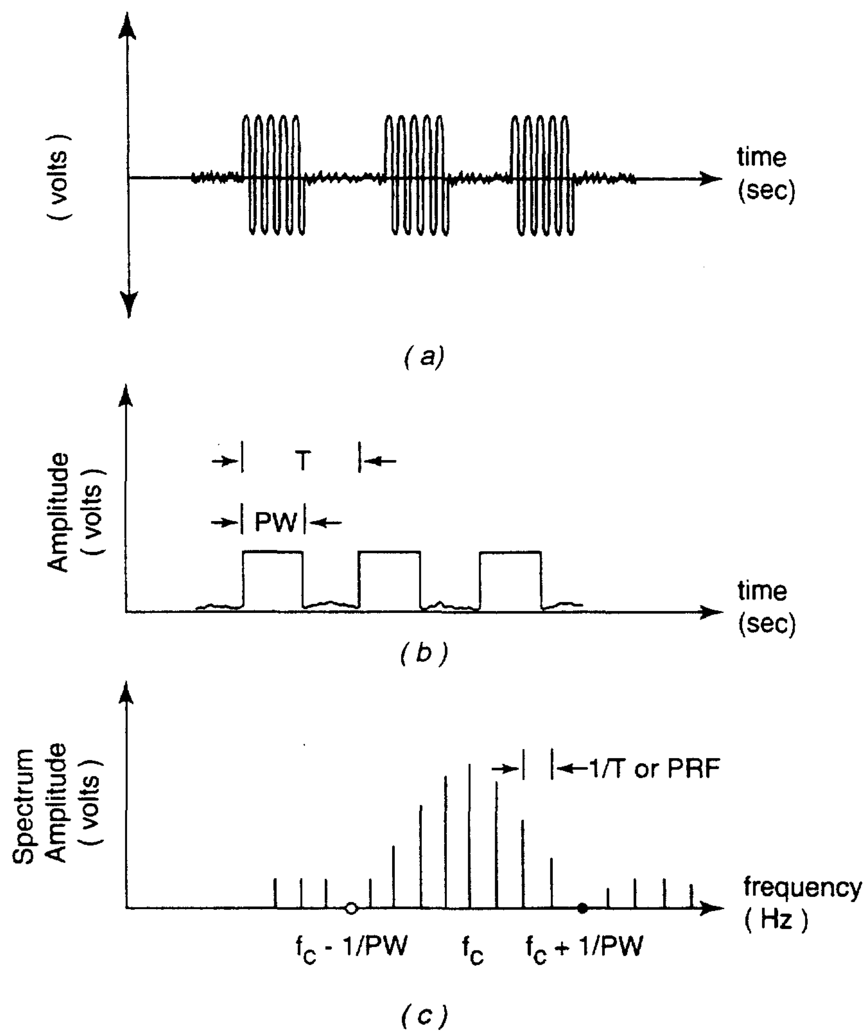

Figure A.9. Periodically pulsed sinusoid signal (a), signal amplitude (b), and amplitude of signal spectrum (c).

The APD of a periodically pulsed sinusoid is dependent on the receiver center frequency, band limiting filter parameters, and pulse parameters. *Pulse overlap distortion* is significant until the *band limited pulse* bandwidth (BW) exceeds the PRR. The band limited pulse is the pulse present at the output of the receiver filter. Analytically the band limited pulse is obtained by convolving the pulse shape with the receiver filter impulse response. Band limited pulses with minimal overlap are considered *independent* or *resolved.* The transmitted pulse shape is fairly well preserved when the filter BW is greater than 2/PW. The two graphs in Figure A.10 show a succession of APDs with the different receiver center frequency and BW combinations listed in Table A.1. The first graph has filter BWs less than the PRR while the second graph has filter BWs greater than the PRR.

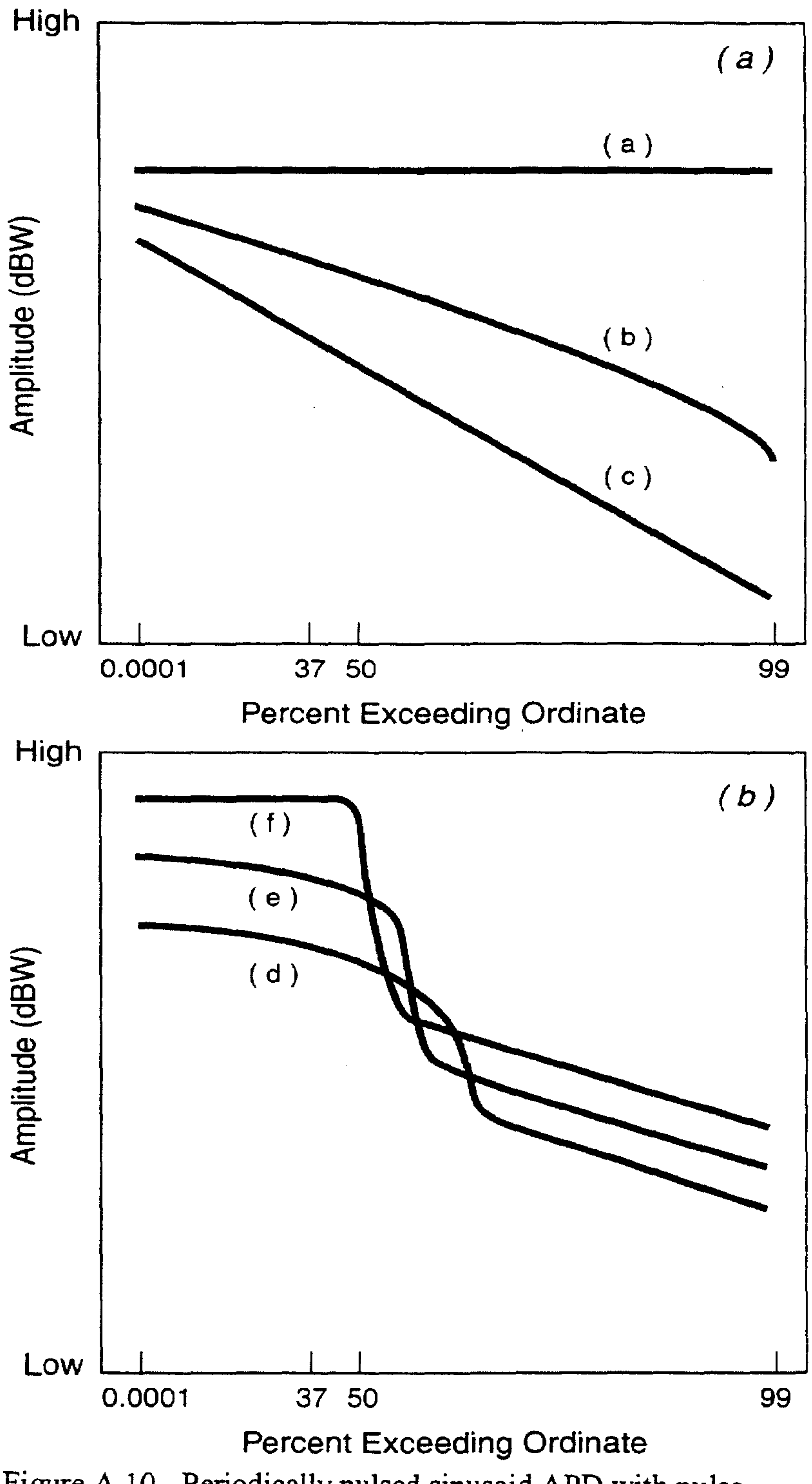

Figure A.10. Periodically pulsed sinusoid APD with pulse repetition frequencies less than (a) and greater than (b) the receiver filter bandwidth. See Table A.1 for receiver center frequency and filter bandwidth conditions for curves a-f.

Table A.1. Figure A.10 APD Conditions

Figure, Curve	**Receiver Center Frequency**	**Bandwidth**
A.10a, a	Tuned to spectral line	Filter BW<< PRR
A.10a, b	Tuned to spectral line	Filter BW< PRR
A.10a, c	Tuned off spectral line	Filter BW< PRR
A.10b, d	Tuned to pulse center frequency	Band limited pulse BW>PRR
A.10b, e	Tuned to pulse center frequency	Band limited pulse BW>>PRR
A.10b, f	Tuned to pulse center frequency	Band limited pulse BW > 2/PW

The APD takes on three characteristics when the filter bandwidth is less than the PRR, as shown in Figure A.10a. If the center frequency is tuned to a spectral line frequency and the filter bandwidth is able to *resolve* the line, it has a sinusoid APD (a). If the center frequency is tuned to a spectral line frequency, but the filter bandwidth is wider than necessary to resolve the line, it can have a Rician APD (b). If the center frequency is tuned to avoid a spectral line frequency, it has a Rayleigh APD (c).

Pulse overlap distortion decreases as the band limited pulse BW increases beyond the PRR as shown in Figure A.10b. The APDs are clearly non-Gaussian. The APD is somewhat curved at the lower probabilities for narrow filter bandwidths where there is pulse overlap (d). The APD flattens at low probabilities for wider filter bandwidths where the pulse overlap is minimal (f).

The low probability amplitudes correspond to the band limited pulse amplitudes. The high probability amplitudes correspond to the receiver noise amplitudes. The amplitudes at low probabilities are proportional to filter BW corresponding to a '$20\log_{10}$ bandwidth rule'. The amplitudes at high probabilities are proportional to the square root of filter BW corresponding to the '$10\log_{10}$ bandwidth rule'. The transition probability between these two domains is related to the band limited pulse duty cycle.

A.3.4 Summary Table

Table A.2. summarizes the APD dependencies for the three tutorial signals.

Table A.2. Tutorial Signal APD Dependencies

Signal	**Receiver Center Frequency**	**Receiver Filter**	**Other Parameters**
WGN	No	BW	
Sinusoid with WGN	Yes	BW	
Periodically-pulsed sinusoid with WGN	Yes	BW	PW, PRR

A.4 Band Limited UWB APDs

A.4.1 UWB Signals

The UWB signal is a train of pulses whose widths (in time) are "ultrashort" and bandwidths (in frequency) are "ultrawide". Like the periodically pulsed sinusoid, the pulses are defined by a PW and PRR. Unlike the periodically pulsed sinusoid, the impulses do not modulate a carrier frequency prior to being transmitted.

For some applications the pulse train may be pulse position modulated by a *time-dither sequence*. Time-dithering attenuates the discrete spectral line PSD component caused by periodic pulse transmission and introduces a continuous, random noise PSD component. The effectiveness of dithering is dependent on time-dither characteristics such as the distribution of dithering times, the reference time of the time-dithered pulse (absolute or relative to the last pulse), and the length of the time-dither sequence.

UWB signals are used in radar and communication devices. These devices reduce power requirements and alleviate spectral congestion by "gating" the pulse train off when continuous transmissions are not needed. They also use uncorrelated dither sequences to minimize interference to other UWB devices operating in the same room or building.

Figure A.11 shows a UWB undithered pulse train, a dithered pulse train, and a gated and dithered pulse train. Figure A.11 also shows an example UWB signal PSD with continuous and discrete components.

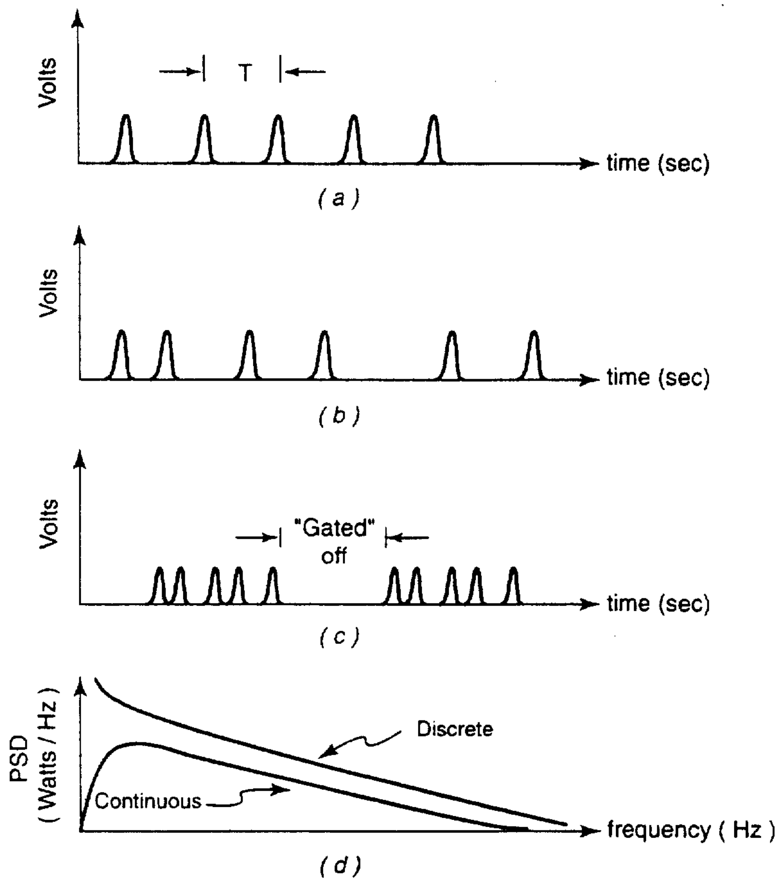

Figure A.11. Undithered (a), dithered (b), and dithered and gated (c) ultawideband signal. Dithered ultrawideband signal power spectral density (d) showing discrete and continuous components. The discrete components are represented as a curve because the lines cannot be resolved graphically.

A.4.2 Band Limited UWB Signals

The bandwidth of the interfering UWB signal is typically several orders of magnitude wider than that of the band limiting filters in the victim narrowband receiver. Thus the pulse shape and BW of the band limited pulse corresponds to the impulse response and BW of the receiver filter. Pulses are independent or resolved when the filter BW is greater than the PRR. Pulses that were independent or resolved before dithering may not be when dithering is introduced. To remain resolved, the pulse repetition period must be greater than the sum of the pulse duration and the maximum dither time.

Band limiting can occur in several of the narrowband receiver functions including demodulation, detection, and signal parameter estimation. Signal parameter estimation is necessary to provide frequency, phase, amplitude, and timing information to the demodulation and detection functions. The bandwidths associated with each of these functions may differ by several orders of magnitude. The relationships among these functions are shown in Figure A.12.

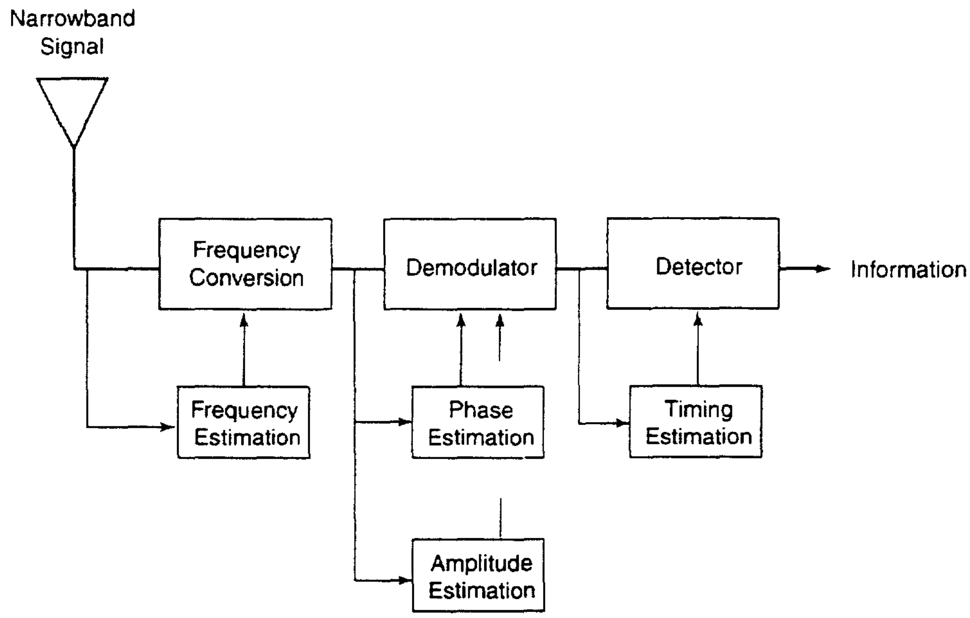

Figure A.12. Locations of band limiting filters in narrowband receivers.

A.4.3 Band Limited UWB Signal APDs

The undithered UWB signal APD will behave similarly to the periodically pulsed sinusoid APD as the filter bandwidth is varied from less than the PRR towards filter bandwidths much greater than the PRR. The dithered UWB signal APD will also behave similarly to the periodically pulsed sinusoid APD as long as the dithered pulses remain resolved. Figure A.13 shows an example of the changes that might happen to an unresolved dithered UWB signal APD when dithering is varied and BW is constant. These effects of dithering are only one possibility among many which are dependent on frequency, dithering distribution, dither reference time, length of dither sequence, gating, modulation, and filtering. In filter bandwidths less than the PRR increased dithering caused this APD to progress from the sinusoid APD to the Rician APD and finally to the Rayleigh APD. The receiver center frequency in this case was tuned to a spectral line. This progression is illustrated in Figure A.13a. In filter bandwidths comparable to the PRR, increased dithering caused this APD to progress from the non-Gaussian noise APD towards the Gaussian noise APD with Rayleigh distributed amplitudes. This progression is illustrated in Figure A.13b.

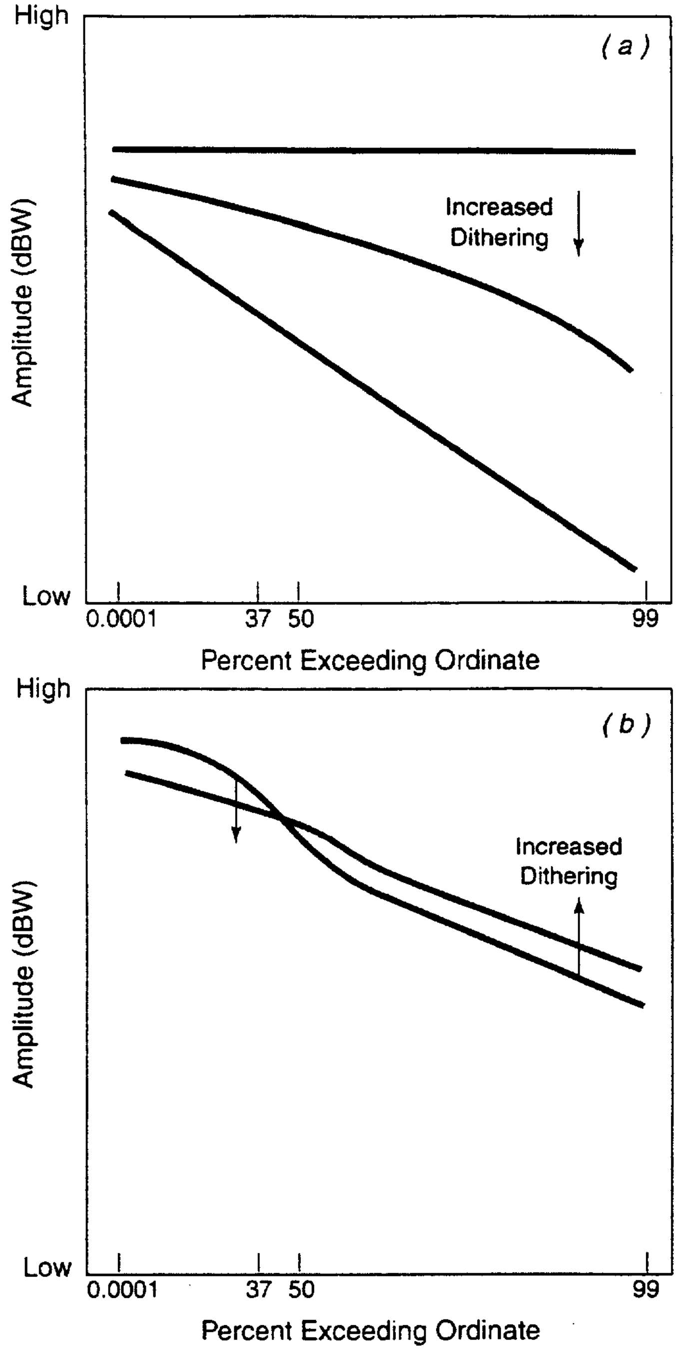

Figure A.13. Effects of increased dithering when band limiting filter bandwidth is less than (a) and comparable to (b) the pulse repetition frequency.

A.5 APD Special Topics

A.5.1 APD Measurement

Spectrum analyzer measurements can be used to estimate the APD or an amplitude statistic such as peak voltage. A block diagram of a spectrum analyzer is shown in Figure A.14. The received signal is converted to an intermediate frequency, band limited by the variable resolution bandwidth filter, and compressed by the log amplifier. Compression by the log amplifier extends the dynamic range of the measurement. The envelope detector extracts the amplitude from the band limited and compressed signal. The video bandwidth filter is used to (video) average the amplitude. The peak detector holds the highest amplitude since it was last reset.

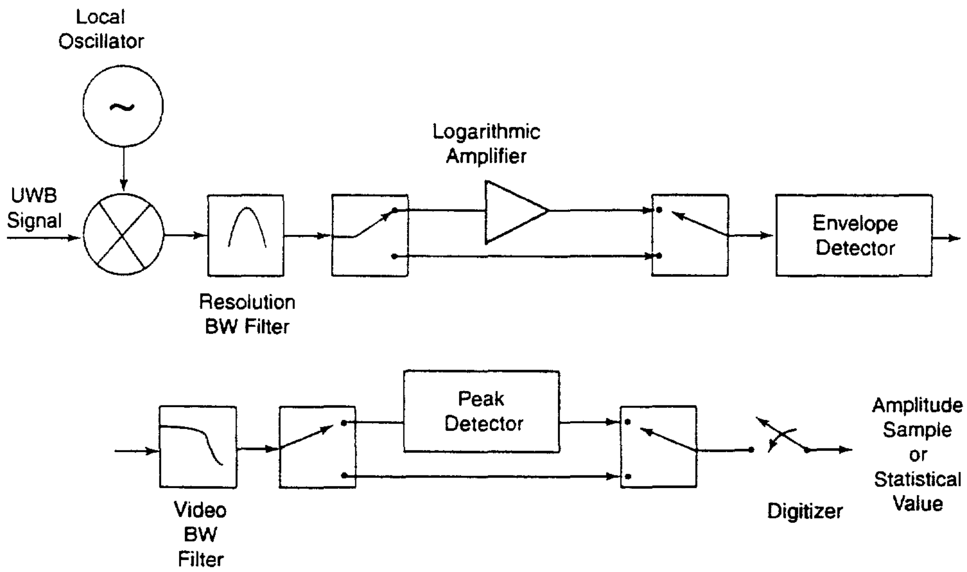

Figure A.14. Spectrum analyzer block diagram.

The statistics of the APD are derived from independent amplitude measurements. The amplitude measurements are considered independent if the sample time is 5 to 10 times the resolution bandwidth. The peak detector is bypassed and video averaging is disabled during an APD measurement. A histogram of the amplitude measurements is used to estimate the APD as shown in Section 2.1.

The peak voltage statistic is measured with the peak detector. Video averaging is disabled during peak detection. Average voltage statistics are measured with the video bandwidth filter. The log amplifier is bypassed and the peak detector is disabled for this measurement. The integration-time of the video bandwidth filter is long enough to suppress variation but surely more than the reciprocal of the resolution bandwidth. Average logarithm voltage statistics are measured in the same manner as the average voltage; however, the signal is log amplified.

A.5.2 APD of the Sum of Band Limited UWB Signals

APDs of band limited UWB signals are often collected individually in a laboratory setting. These APDs are useful for studying the interference of one UWB signal. However, in everyday life, more than one UWB device may be transmitting at a time. The statistics of the aggregate signal are conditional on the distributions of the individual band limited UWB signals and the number of signals that are to be added. Assuming the phases of the band limited UWB signals are uniformly distributed, four cases of interest emerge as shown in Table A.3.

Table A.3. Distributions of Four Aggregate APD Cases

	Distributions of band limited UWB signals	
Number of UWB signals	**Rayleigh**	**Any Distribution**
Few	(1) Rayleigh	(3) Non-Rayleigh
Many	(2) Rayleigh	(4) Rayleigh

In case 1 and 2 all the band limited UWB signals have Rayleigh distributions and the aggregate is also Rayleigh distributed. Case 4 is Rayleigh distributed by virtue of the central limit theorem of statistics. In cases 1,2, and 4 the aggregate power is simply the sum of the individual UWB signal powers. Measurement system average noise power can be removed from individual APDs before summing. In addition the average power of the individual APD may have to be reduced by attenuation due to the propagation channel to compensate for differences in location.

Case number 3 is more difficult for two reasons. First measurement system noise statistics cannot be removed from the measured statistics. Second, the computation of the aggregate APD requires using the joint statistics of a band limited UWB signal amplitude and phase distributions. For these reasons it is best to measure these statistics as an aggregate.

A.5.3 Performance Prediction

Characterizing the band limited UWB signal with an APD is not enough. Ultimately the effect that the amplitude statistics have on victim receiver performance has to be determined. The band limited UWB interference takes three guises: sinusoidal interference, Gaussian noise with Rayleigh distributed amplitudes, and non-Gaussian noise. The APD is particularly useful for predicting the performance of non-Gaussian noise.

For example, Figure A.15 shows how the non-Gaussian noise APD can be used to predict bit error rate (BER) for non-coherent binary frequency-shift keying. The straight, sloped APD is the result of band limited Gaussian noise produced in the receiver. The stepped APD is the result of band limited non-Gaussian interference. The average receiver noise power is represented by a horizontal line on the graph. The signal-to-noise ratio (SNR) is represented as another horizontal line SNR dB above the noise power line. The BER is approximately one-half the probability where the SNR line intersects the APD curve.

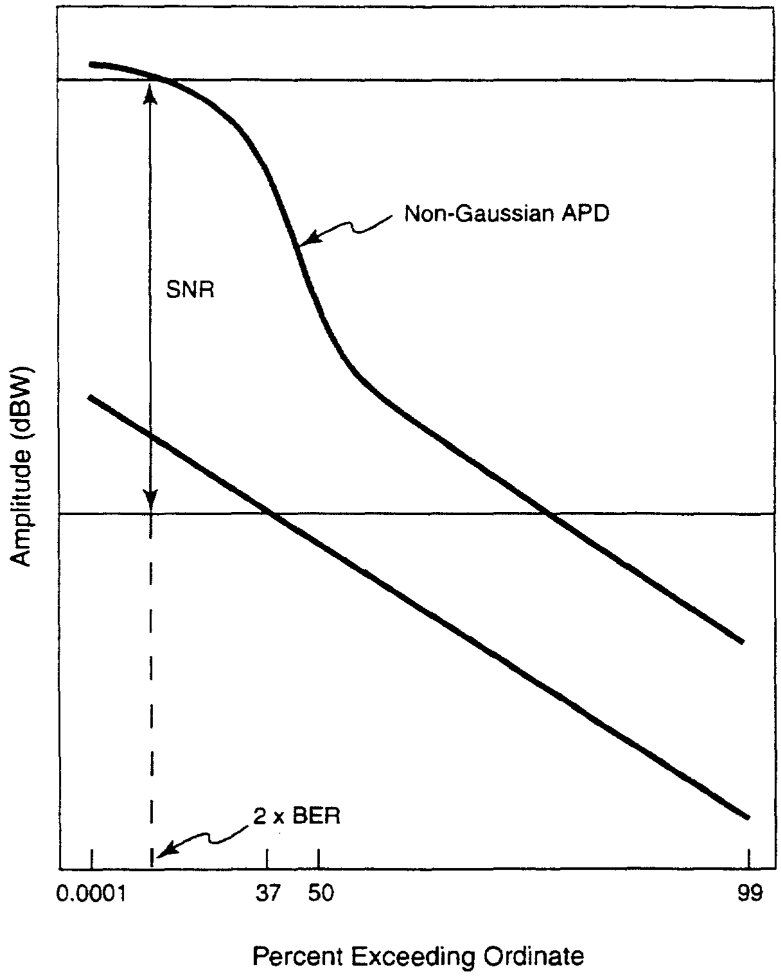

Figure A.15. Estimation of bit error rate from a non-Gaussian noise APD.

Many modern digital receivers use elaborate error correction and time-interleaving techniques to correct errors in the received bit sequence. In such receivers, the corrected BER delivered to the user will be substantially different from the received BER. Computation of BERs in these receivers will require much more detailed interference information than is contained in the APDs. For example, second-order statistics of noise amplitudes describing the time of arrival of noise amplitudes may be needed.

A.6 Concluding Remarks

This tutorial has shown that the APD is a useful tool for describing the UWB signal and analyzing UWB signal interference to victim narrowband receivers. It is useful because it 1) is measurable, 2) provides a variety of statistical values, and 3) can be used to aid in receiver performance prediction.

The APD gives insight to the potential interference from UWB signals in a wide variety of receiver bandwidths and UWB characteristics, especially when the combination of interferer and victim produces non-Gaussian interference in the victim receiver. If the interference is Gaussian, victim receiver performance degradation is correlated to the interfering signal average power alone and there is no need for further analysis using the APD. If the interference is non-Gaussian or sinusoidal, information in the APD may be critical to quantifying its effect on victim receiver performance degradation. Band limited UWB interference becomes increasingly non-Gaussian as the victim narrowband receiver bandwidth increases beyond the UWB signal PRR. Band limited UWB interference becomes increasingly sinusoidal as the victim narrowband receiver bandwidth decreases below the UWB signal PRR and a spectral line is present within the receiver bandwidth.

Spectrum regulators frequently use amplitude statistics such as peak, RMS, or average logarithm voltage to regulate transmitters. Further work is needed to determine if these statistics are strongly correlated to narrowband receiver performance. If these statistics are not correlated to receiver performance, it may be necessary to set regulatory limits in terms of the APD itself.

A.7 Bibliography

[1] A.D. Spaulding and R.T. Disney, "Man-made radio noise, part 1:Estimates for business, residential, and rural areas," OT Report 74-38, Jun. 1974.

[2] A.D. Spaulding, "Digital system performance software utilizing noise measurement data," NTIA Report 82-95, Feb. 1982.

[3] A.D. Spaulding,"The natural and man-made noise environment in personal communication services bands," NTIA Report 96-330, May 1996.

[4] R.J. Achatz, Y. Lo, P.B. Papazian, R.A. Dalke, and G.A. Hufford, "Man-made noise in the 136-138-MHz VHF meteorological satellite band," NTIA Report 98-355, Sep. 1998.

[5] R.A. Dalke, "Radio Noise," in the *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 18., J.G. Webster, Ed.,John Wiley & Sons, Inc., New York: N.Y., 1998, pp. 128-140.

[6] R.J. Matheson, "Instrumentation problems encountered making man-made electromagnetic noise measurements for predicting communication system performance," *IEEE Transactions on Electromagnetic Compatibility*, Volume EMC-12, Number 4, Nov. 1970.

[7] J.S. Bendat and A.G. Piersol, *Measurement and Analysis of Random Data*, New York, NY: John Wiley & Sons, Inc, 1966.

[8] P.L. Meyer, *Introductory Probability and Statistical Applications*, Second Edition, Reading, Mass: Addison-Wesley Publishing Company, 1970.